AN OPTIMAL SETTING FOR PRODUCTION PROCESS USING EXPECTED INVERTED LOSS FUNCTION

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***Abstract*—** In manufacturing, loss Function expresses the economic consequences associated with deviation from target; since different process have different sets of economic consequences a better approach to developing loss function and setting optimal values is desirable. This work focused on the use of loss function based on inverted normal probability density Function known as the inverted Normal loss function in determining an optimal setting for a production process. Basic programming language is used to determine the expected loss of weights with varying loss values, on weights of cigarette. The optimal value for the manufacturing of cigarette process was obtained to be 1030mg.

Keywords: Loss function, inverted Normal loss function, optimal value, production process

QUALITY AND LOSS FUNCTIONS

It is essential that products meet the requirements of those who use them. Therefore quality is defined as fitness for use. It can also be defined as conformance to the agreed requirement of customers. The term customer applies to those involved in its sales or consumers.

Taguchi defined quality as a loss a product causes to society after being shipped, other than any losses caused by its intrinsic function. This definition can very well be taken as a definition of “non-quality” rather than of “quality”. But the essence of these words lies in the fact that Taguchi is opposed to treating quality question as “value questions”. He believes that the desirability of a product is determined by the societal loss it generates from the time it is shipped to the customer; the smaller the loss, the higher the desirability. (Logothetis and Wynn (1989)

Variations from desired functional specification cause loss of quality. There is a direct loss due to warranty and increased service costs and to dissatisfied customers. There is also an indirect loss due to market share loss and to increase marketing efforts to overcome competitiveness. An appropriate quality improvement programmed should have as a main objective the minimization of variation of product performances about their target values. The smaller the performance variation the better the quality and the larger the deviation from the target the larger the loss.

LOSS FUNCTION

In decision theory, loss is generally defined as a function of the deviation of an estimator from the parameter value to be estimated. Loss functions are used in Decision Theory application and Quality Assurance settings, to quantify losses associated with deviation from a desired target value. It is also used in quality assurance setting, to reflect the economic loss associated with variation and deviations from the process target or target value of a product characteristic (Spiring and Yenng (1998).

The most common loss function is the quadratic functions corresponding to a Gaussian noise model with zero mean, and a standard deviation that does not depend on the inputs. The Gaussian loss function is used because it has nice analytical properties. However, one of the potential difficulties of the quadratic loss function is that it receives large contributions from outliers that have particularly large errors. If there are long tails on the distributions then the solution can be dominated by a very small number of outliers. The techniques that attempt to solve this problem are referred to as robust statistics (Huber, 1972)

Taguchi (1986) used a modified quadratic loss function to assess and illustrate losses associated with deviations of a product characteristic from target. This loss function takes the following basic quadratic form

$$L(X) = k(x-m)^2$$

- Where x is a measure of the characteristics, L is the loss in monetary terms, m is the point at which the characteristic is actually set, and k is a constant that depends on the magnitude of deviation from target.
- In response to criticisms of the quadratic loss function, Spiring (1993) proposed a loss function based on the inversion of normal probability density function.
- The resulting Inverted Normal loss function (INLF) differs from the traditional quadratic loss, in that it is bounded and provides a more reasonable assessment of loss associated with deviation from target. Sun, Laramee and Remberg (1996) improved on the inverted normal loss (INLF) further.
- The quality characteristics of cigarette measured by the quality control department of an International Tobacco company are weight, circumference, length and pressure drop. The weight of cigarette hereafter denoted by x is focused in this work.
- Consider a situation where each stick of cigarette has a target weight of 1000mg. A stick of cigarette must be reprocessed if it is under- weighs, while those on or above the target weight are sent directly to the market. Under-weight therefore attracts more economic loss to the producer than over-weight. The economic loss around the target therefore is asymmetric. Under weight therefore attracts more economic loss to the producer than over weight. However the normal loss function is adopted for this work because the data collected is approximately normally distributed.

GENERAL CLASS OF LOSS FUNCTION

The general class of loss function is based on the inversion of common probability density functions. These classes of loss functions satisfy the criteria that the loss must always be positive, minimum at the target value, monotonically increasing as the process deviates from target and reaches a quantifiable maximum.

Let g (x, T) denote the probability density function (pdf) used in creating the economic loss function for the process and f (x, $\hat{\theta}$) the statistical distribution associated with the process measurements.

f(x, $\hat{\theta}$) when $\hat{\theta} = \{\mu, \sigma^2\}$ is the statistical distribution of the process under study, which is normally distributed with mean μ_L and variance $\sigma^2{}_L$. Where μ_L and σ_L refer to the parameter base on the sample taken from the process.

The general form of the inverted probability loss function (IPLF) is define to be

$$L(x,T) = k\left(1 - \frac{g(x,T)}{m}\right) \qquad \forall\ x \in \Omega \quad \text{-----------------------(1)}$$

Where x denotes the process measurement, k the maximum loss, Ω the measurement space and T, the process target. If f (x, $\hat{\theta}$) denotes the probability density function associated with the behavior of process measurement x, the general form of the expected loss function associated with equation (1) will be

$$E(L(x,T)) = \int_{\Omega} k\left(1 - \frac{g(x,T)}{m}\right) f\left(x,\hat{\theta}\right) dx$$

$$= k\int_{\Omega} 1 - \frac{g(x,T)}{m} . f\left(x,\hat{\theta}\right) dx$$

$$= k\left(1 - \frac{1}{m}\right)\int_{\Omega} g\left(x,T\right). f\left(x,\hat{\theta}\right) dx$$

INVERTED NORMAL LOSS FUNCTION

Consider a normal probability density function (pdf) to define g(x,T):

$$g(x,T) = \frac{1}{\sigma_L \sqrt{2\pi}} \exp - \left(\frac{(x-T)^2}{2\sigma^2{}_L} \right) \qquad -\infty<x<\infty$$

where T denotes the target and σ_L denotes a scale parameter and the supremum of

g (x, T) in this case is $m = \frac{1}{\sigma_L \sqrt{2\pi}}$

Since $L(x,T) = k\left(1 - \frac{g(x,T)}{m}\right)$ in (1)

The inverted normal loss function then becomes

$$L(x,T) = k\left(1 - \frac{\frac{1}{\sigma_L\sqrt{2\pi}} * \exp - \frac{(x-T)^2}{2\sigma^2{}_L}}{\frac{1}{\sigma_L\sqrt{2\pi}}}\right)$$

Therefore, $L(x,T) = k\left(1 - \exp - \frac{(x-T)^2}{2\sigma^2{}_L}\right)$ $-\infty < x < \infty$

In the case of an asymmetric situation for the general class of loss function

$$L(x,T) = \begin{cases} k_1\left(1 - \frac{g(x,T)}{m}\right) & \forall x < T \\ k_2\left(1 - \frac{g(x,T)}{M}\right) & \forall x \geq T \end{cases}$$

And for the inverted normal loss with an asymmetric situation

$$L(x,T) = \begin{cases} k_1\left(1 - \exp - \left(\frac{(x-T)^2}{2\sigma^2{}_L}\right)\right) & \forall x \in (0,T) \\ k_2\left(1 - \exp - \left(\frac{(x-T)^2}{2\sigma^2{}_L}\right)\right) & \forall x \in (T,\infty) \end{cases}$$

EXPECTED LOSS FOR INVERTED NORMAL LOSS FUNCTION

$$E[L(x,T)] = \int_{-\infty}^{\infty} k\left(1 - \exp - \left\{\frac{(x-T)^2}{2\sigma^2{}_L}\right\}\right) * f(x,\hat{\theta})dx$$

For an asymmetric case

$$E[L(x,T)] = \int_{-\infty}^{T} k_1\left(1 - \exp - \left\{\frac{(x-T)^2}{2\sigma^2_L}\right\}\right) * f(x,\hat{\theta})dx + \int_{T}^{\infty} k_2\left(1 - \exp - \left\{\frac{(x-T)^2}{2\sigma^2_L}\right\}\right) * f(x,\hat{\theta})dx$$

$$= \int_{-\infty}^{T} k_1 \left(1 - \exp - \left\{ \frac{(x-T)^2}{2\sigma^2_L} \right\} \right) . \frac{1}{\sigma_L \sqrt{2\pi}} \exp - \left\{ \frac{(x-\mu_L)^2}{2\sigma_L} \right\} dx$$

$$+ \int_{T}^{\infty} k_2 \left(1 - \exp - \left\{ \frac{(x-T)^2}{2\sigma^2_L} \right\} \right) . \frac{1}{\sigma_L \sqrt{2\pi}} \exp - \left\{ \frac{(x-\mu_L)^2}{2\sigma_L} \right\} dx$$

$$= \int_{-\infty}^{T} k_1 \frac{1}{\sigma_L \sqrt{2\pi}} \exp - \left\{ \frac{(x-\mu_L)^2}{2\sigma^2_L} \right\} - k_1 \exp - \left\{ \frac{(x-T)^2}{2\sigma^2_L} \right\} . \frac{1}{\sigma_L \sqrt{2\pi}} \exp - \left\{ \frac{(x-\mu_L)^2}{2\sigma^2_L} \right\} dx$$

$$= k_1 \int_{985}^{1000} \frac{1}{\sqrt{2\pi}\sigma_L} \exp \left\{ - \frac{(x-T)^2}{2\sigma^2_L} \right\} * \exp \left\{ - \frac{(x-\mu_L)^2}{2\sigma^2_L} \right\} dx$$

$$+ \int_{T}^{\infty} k_2 \frac{1}{\sigma_L \sqrt{2\pi}} \exp - \left\{ \frac{(x-\mu_L)^2}{2\sigma^2_L} \right\} - k_2 \exp - \left\{ \frac{(x-T)^2}{2\sigma^2_L} \right\} . \frac{1}{\sigma_L \sqrt{2\pi}} \exp - \left\{ \frac{(x-\mu_L)^2}{2\sigma^2_L} \right\} dx$$

$$- k_1 \int_{T}^{\infty} \frac{1}{\sigma_L \sqrt{2\pi}} \exp - \left(\frac{(x-T)^2}{2\sigma^2_L} \right) . \exp - \left(\frac{(x-\mu_L)^2}{2\sigma^2_L} \right) dx$$

This is used in determining the optimal value

THE OPTIMAL SETTING FOR THE CIGARETTE PRODUCTION PROCESS.

In order to reduce reprocessing of cigarette by the manufacturer, the machine needs to be set in such a way that the weight of cigarette being produced will always be on or above the set target values because under-weight is more economically consequential than over-weight to the producer. Hence it is worthwhile to have an optimal setting.

To set an optimal value for the production process the expected loss for this process is given by

$$= k_1 \phi \left(\frac{(T - \mu_L)}{\sigma_L} \right) + k_2 \left(1 - \phi \left(\frac{(T - \mu_L)}{\sigma_L} \right) \right)$$

$$- k_2 \int_{T}^{\infty} \frac{1}{\sigma_L \sqrt{2\pi}} \exp - \left(\frac{(x-T)^2}{2\sigma^2_L} \right) . \exp - \left(\frac{(x-\mu_L)^2}{2\sigma^2_L} \right) dx$$

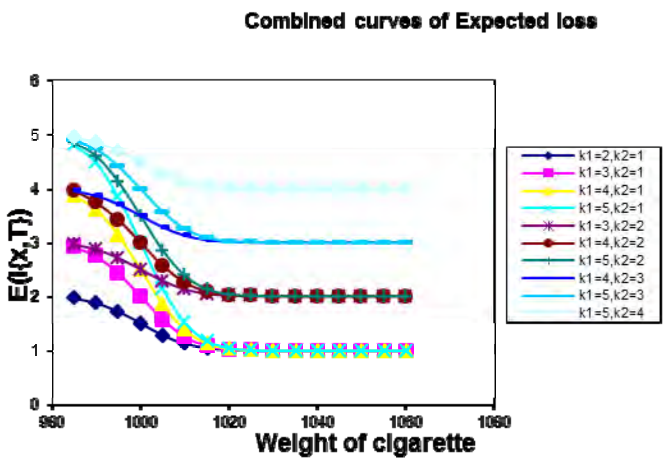
$$\text{E[L(x,T)]} = k_1 \phi \left(\frac{(1000-\mu_L)}{\sigma_L} \right) + k_2 \left(1 - \phi \left(\frac{(1000-\mu_L)}{\sigma_L} \right) \right)$$

$$k_2 \int_{1000}^{1060} \frac{1}{\sqrt{2\pi}\sigma_L} \exp \left\{ - \frac{(x-T)^2}{2\sigma^2_L} \right\} * \exp \left\{ - \frac{(x-\mu_L)^2}{2\sigma^2_L} \right\} dx$$

A BASIC programme is used to compute the associated expected loss {E[L(x,T)]} for weight of cigarette at various combinations of the value of k_1 and k_2 .where k1, is the maximum loss when weight is below the target and k2 the maximum loss when weight is above the target. The resulting expected loss is shown in table 1.0, and the corresponding curve for the expected loss is shown in figure 1.0

TABLE OF THE EXPECTED LOSS DUE TO CIGARETTE PRODUCTION PROCESS

Weight	K1=2 K2=1 E(L(X,T))	K1=3 K2 = 1 E(L(X,T))	K1=4 K2 = 1 E(L(X,T))	K1=5 K1=1 E(L(X,T))	K1=3 K1=2 E(L(X,T))	K1=4 K2 = 2 E(L(X,T))	K1=5 K2 = 2 E(L(X,T))	K1=4 K1=3 E(L(X,T))	K1=5 K2 = 3 E(L(X,T))	K1=5 K2 = 5 E(L(X,T))
985	1.955	2.910	3.865	4.820	2.955	3.970	4.865	3.955	4.910	4.9551
990	1.869	2.739	3.608	4.478	2.869	3.739	4.608	3.869	4.739	4.8696
995	1.709	2.419	3.128	3.838	2.709	3.419	4.128	3.709	4.419	4.7096
1000	1.499	2.000	2.499	3,000	2.500	2.999	3.499	3.500	3.999	4.4996
1005	1.285	1.570	1.853	2.140	2.285	2.570	2.855	3.285	3.570	4.2850
1010	1.127	1.255	1.383	1.511	2.127	2.255	2.383	3.127	3.255	4.1278
1015	1.044	1.088	1.132	1.177	2.044	2.088	2.132	3.044	3.088	4.0443
1020	1.011	1.023	1.034	1.046	2.011	2.023	2.034	3.011	3.023	4.0115
1025	1.002	1.004	1.006	1.009	2.002	2.004	2.006	3.002	3.004	4.0023
1030	1	1	1	1	2	2	2	3	3	4
1035	1	1	1	1	2	2	2	3	3	4
1040	1	1	1	1	2	2	2	3	3	4
1045	1	1	1	1	2	2	2	3	3	4
1050	1	1	1	1	2	2	2	3	3	4
1055	1	1	1	1	2	2	2	3	3	4
1060	1	1	1	1	2	2	2	3	3	4



Using the value of weight and varying the values of k1 and K2, table 1.0 gives the expected loss with the curves given in figure 1.0. The expected loss kept on reducing as the value of weight increases and later converges at various value of k_1. Also from the curves the wider the range k_1 and k_2 the higher the slope of the curves. Their points of convergence then result to the optimal setting.

The process optimal setting that gives the first minimum expected loss is therefore 1030mg

SUMMARY OF RESULTS, CONCLUSION AND RECOMMENDATIONS.

The expected loss for the process provides the optimal setting for this work is 1030mg.

In order to reduce reprocessing by manufacturer, the machines needs to be set such that the quality characteristic of a product being produced will always be on or above the set target values since below target is more economically consequential than over target to the producer. Also for the facts that a shift in process mean may be inevitable the use of optimal setting will go along way in setting the process optimally.

Furthermore, the expected loss functions will continue to provide insights into optimal process settings and tracking opportunity.

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