Modification of recyclable outputs via Modified CCR model

Mehrdad Nabahat
Sama Technical and Vocational Training School
Islamic Azad University, Urmia branch Urmia, Iran
E-mail: m_nteach2009@yahoo.com

Fardin Esmaeeli Sangari
Sama technical and vocational training school
Islamic Azad University, Urmia branch, Urmia, Iran
E-mail: fardin_e_s@yahoo.com

Mohammad Masdari
Department of Computer Engineering,
Islamic Azad University, Urmia Branch, Urmia, Iran,
E-mail: M.Masdari@Iaurmia.ac.ir

***Abstract*—Data Envelopment Analysis (DEA) is a non-parametrical method for evaluating the efficiency of Decision Making Units (DMUs) using mathematical programming. But in special cases for the reason of a few difficulties such as weak efficient frontier and computing of non- Archimedean number and etc. this evaluation cannot do exactly. For evaluating the efficiency of DMUs, some of the units are located on the weak bound. In the facet analysis, we can move the units on weak boundary toward the effective frontier by selecting the lower bound for variables. Previously, some studies have been done to modify weak frontier at basic DEA model. In DEA models it has been assumed that outputs go outside of the system but there are systems in which some part of the outputs possibly used as inputs in the system. In this article we show the coherency between facet analysis of general DEA model and outreached DEA model. Finally, we will introduce the modification of weak frontiers and present the results by a numerical example**.

Keywords-Data envelopment analysis (DEA); Modified CCR.

I. INTRODUCTION

Data Envelopment Analysis (DEA) is a theoretical framework for performance analysis. It is a set of linear programming technique used to construct empirical production frontiers and evaluates the relative efficiency of systems, Decision Making Units (DMUs) with multiple inputs and outputs by given input-output data. Charnes, Cooper, Rhodes [5] (1978) were the pioneers of the field that introduced their first model named “CCR” in 1978 for evaluating the efficiency of DMUs. DEA has demonstrated to be an effective technique for measuring the relative efficiency of a set of homogeneous DMUs which utilize the same inputs to produce the same outputs. In conventional DEA applications, given a set of available measures, it is assumed that the status of each measure is clearly used as an input or output variable in the production process prior to using DEA. In conventional DEA models, it has been assumed that the produced outputs are considered as final outputs. However, in real world, in some situations, some portion of the produced outputs may be considered as inputs to the system. These outputs enter the system as inputs once again and they are referred to as recyclable outputs. So, the system is fed by a mixture of external inputs and recyclable outputs. In this case, recyclable outputs can play input role. Amirteimoori and Khoshandam [3] proposed a model in which they can evaluate the efficiency of the systems with recyclable outputs. For this type of production system, a modified DEA model is proposed to incorporate such outputs. These outputs will be considered as inputs and outputs simultaneously to maximize the relative efficiency of the system.

Cook and Zhu [6] proposed a method for classifying input and output variables. They considered variables whose status is flexible. These measures can play either input or output roles. They presented a modification of the standard DEA model to accommodate flexible measures. In 1979 Charnes, Cooper and Rohdes [7] provided a short communication in which non-Archimedean number ε has been used as a lower bound of factor weights to show inefficiency of weak efficient DMUs.

Daneshvar method [1] tries to show that considering a unique ε as a lower bound of all factor weights cannot compute the correct efficiency scores for weak efficient DMUs and DMUs, which are compared with them. Then a method will be provided to compute a lower bound for each factor weights. These values are used as lower bound of factor weights in CCR model. . Amirteimoori and Emrouznejad [4] proposed a model in which flexible measures are axiomatically imported in a mixed integer linear programming model. Amirteimoori and Khoshandam [2] proposed a model in which each flexible measure is treated as either input or output to maximize the technical efficiency of the DMU under evaluation.

In this paper we want to modify the efficiency of recyclable outputs by choosing the boundary for variables via modified CCR model.

II. MODIFICATION OF RECYCLABLE OUTPUTS USING MODIFIED CCR MODEL

Suppose we have $n\ DMUs$, and that each $DMU_j\ : j = 1, \dots, n$ uses m inputs $x_{ij}\ i = 1, \dots, m$ to produce two types of outputs: $y_{rj}\ :\ r = 1, \dots, s$ and $z_{kj}\ : k = 1, \dots, t$. The outputs y_{rj} are final outputs, but the outputs z_{kj} are recyclable and they can be entered to the system once again. So, the system is fed by a mixture of external inputs x_{ij} and the recyclable outputs z_{kj}.

The efficiency measure for DMU_o is defined as:

$$\begin{aligned} Max \quad & \sum_{r=1}^{s} u_r\, y_{ro} + \sum_{k=1}^{t} w_k d_k\, z_{ko} \\ st. \quad & \sum_{i=1}^{m} v_i\, x_{io} + \sum_{k=1}^{t} w_k (1 - d_k)\, z_{ko} = 1 , \qquad (1) \\ & \left(\sum_{r=1}^{s} u_r\, y_{rj} + \sum_{k=1}^{t} w_k d_k\, z_{kj} \right) - \left(\sum_{i=1}^{m} v_i\, x_{ij} + \sum_{k=1}^{t} w_k (1 - d_k)\, z_{kj} \right) \le 0 , \\ & u_r, v_i, w_k, \mu_k \ge 0 \\ & 0 \le d_k \le 1 \end{aligned}$$

The efficiency ratio ranges between zero and one, since d_k and w_k are decision variables, model (3) is clearly nonlinear. It can be linearized by using the changes of variables

$w_k d_k = \mu_k , w_k(1 - d_k) = w_k - \mu_k : k = 1, \dots, t\ \ , 0 \le \mu_k \le w_k$. By considering these changes of variables we replace (3) by the following linear program:

$$\begin{aligned} Max \quad & \sum_{r=1}^{s} u_r\, y_{ro} + \sum_{k=1}^{t} \mu_k\, z_{ko} \\ st. \quad & \sum_{i=1}^{m} v_i\, x_{io} + \sum_{k=1}^{t} (w_k - \mu_k)\, z_{ko} = 1 , \qquad (2) \\ & \left(\sum_{r=1}^{s} u_r\, y_{rj} + \sum_{k=1}^{t} \mu_k\, z_{kj} \right) - \left(\sum_{i=1}^{m} v_i\, x_{ij} + \sum_{k=1}^{t} (w_k - \mu_k)\, z_{kj} \right) \le 0 , \\ & 0 \le \mu_k \le w_k \ , \quad k = 1, \dots, t \\ & u_r, v_i, w_k, \mu_k \ge 0 \end{aligned}$$

The efficient DMUs, in which the optimal value of above problem is nonzero, are those that can be located on the intersection of the efficient frontier and the weak efficient frontier hyper planes. Let the set of these DMUs be called β. Now for the DMUs belonging to β solve the following problems:

$$\begin{aligned} Max \quad & v_i \qquad (3) \\ s.t \quad & \sum_{i=1}^{m} v_i x_{io} = 1 \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \qquad j = 1, 2, \dots, n \\ & v_i \ge 0 \qquad i = 1, 2, \dots, m \\ & u_r \ge 0 \qquad r = 1, 2, \dots, s \end{aligned}$$

And

$$\begin{aligned} Max \quad & u_r \qquad (4) \\ s.t \quad & \sum_{i=1}^{m} v_i x_{io} = 1 \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \qquad j = 1,2,\dots,n \\ & v_i \ge 0 \qquad i = 1,2,\dots,m \\ & u_r \ge 0 \qquad r = 1,2,\dots,s \end{aligned}$$

Suppose that the optimal values for (3) and (4) are represented by v_i^+ and u_r^+ respectively. However for each $r = 1,\dots,s$ and $i = 1,\dots,m$ suppose that:

$$\varepsilon_r = Min\{u_r^+ \mid DMU \in \beta\} \;\; \forall r = 1,2,\dots,s \qquad (5)$$

$$\varepsilon_i = Min\{v_i^+ \mid DMU \in \beta\} \;\; \forall i = 1,2,\dots,m \qquad (6)$$

Now based on (5) and (6), model (2) is modified as follow:

$$\begin{aligned} Max \quad & \sum_{r=1}^{s} u_r y_{ro} + \sum_{k=1}^{t} \mu_k z_{ko} \\ st. \quad & \sum_{i=1}^{m} v_i x_{io} + \sum_{k=1}^{t} (w_k - \mu_k) z_{ko} = 1, \qquad (7) \\ & \left(\sum_{r=1}^{s} u_r y_{rj} + \sum_{k=1}^{t} \mu_k z_{kj}\right) - \left(\sum_{i=1}^{m} v_i x_{ij} + \sum_{k=1}^{t} (w_k - \mu_k) z_{kj}\right) \le 0, \\ & 0 \le \mu_k \le w_k, \quad k = 1,\dots,t \\ & u_r \ge \varepsilon_r, v_i \ge \varepsilon_i, w_k \ge 0, \mu_k \ge 0 \end{aligned}$$

III. NUMERICAL EXAMPLE

We consider a group of 25 DMUs with two inputs x_1 and x_2 and four outputs y_1, y_2, z_1, z_2 presented in Table I. The first seven columns of the table show the input-output data. Two outputs z_1, z_2 (columns 6 and 7) are recyclable and they should be sent to the system.

For efficient DMUs we use models (3) and (4) to catch lower bound for variables u_r, v_i that represented in table II.

Table I. Data

j								1 -		**1 $-d_{2j}$**	
1	11	34	141	98	1304	1215	0.4959	0.5041	0.4991	0.5009	0.9853
2	19	43	139	174	1485	1457	0.4917	0.5083	0.4963	0.5036	0.9748
3	21	26	121	172	1251	1325	0.2582	0.7418	0.6282	0.3718	1
4	18	56	168	251	1940	1874	0.5049	0.4951	0.4414	0.5586	0.9919
5	17	41	177	254	2196	2147	0.506	0.494	0.4414	0.5586	1
6	21	44	151	122	2967	2354	0.4889	0.5111	0.4991	0.5009	0.9506
7	19	87	249	238	3298	1369	0.5093	0.4907	0.4844	0.5156	1
8	11	12	131	143	2776	1230	0.5093	0.4909	0.4772	0.5228	1
9	21	90	221	154	1391	1089	0.4444	0.5556	0.4958	0.5042	0.9797
10	14	23	384	162	2353	1981	0.4479	0.5521	0.5106	0.4894	1
11	12	29	339	121	3293	1489	0.5093	0.4907	0.4860	0.5140	1
12	28	51	347	141	4781	1746	0.5883	0.4117	0.4265	0.5735	1
13	19	78	128	131	5215	1654	0.5883	0.4117	0.4272	0.5728	1
14	21	89	136	117	2269	2032	0.4749	0.5251	0.4997	0.5003	0.9516
15	25	65	294	186	1392	2125	0.3825	0.6175	0.5590	0.4410	1
16	21	44	251	189	1154	1258	0.4431	0.5569	0.4962	0.5038	0.9951
17	22	55	349	288	1474	1789	0.4431	0.5569	0.4943	0.5057	1
18	55	19	231	243	1456	1444	0.4431	0.5569	0.479	0.5210	1
19	52	91	321	264	1325	1124	0.4431	0.5569	0.4834	0.5166	1
20	28	28	484	162	1789	1747	0.3879	0.6121	0.4987	0.5013	1
21	43	32	239	191	2100	1369	0.6201	0.3799	0.396	0.6040	0.9723
22	21	33	547	161	2541	1585	0.5752	0.4248	0.4160	0.5840	1
23	29	17	628	151	2315	1364	0.5752	0.4248	0.427	0.5728	1
24	39	29	536	127	2478	1187	0.5734	0.4266	0.4453	0.5547	1
25	48	39	394	206	3258	1587	0.5703	0.4297	0.2380	0.7620	0.9951

Table II. Results of model (3) and (4)

U_2^+	U_1^+	V_2^+	V_1^+	DMU
0.00581395	0.0025174	0.0384615	0.047619	**3**
0.00393701	0.00222297	0.0243902	0.588235	**5**
0.00420168	0.00216841	0.0114943	0.0526316	**7**
0.00699301	0.00372383	0.0833333	0.09090901	**8**
0.00529193	0.00260417	0.0434783	0.0714286	**10**
0.00574628	0.00295835	0.0344828	0.083333	**11**
0.00323014	0.00149332	0.0196078	0.0357143	**12**
0.00385846	0.00197163	0.0128205	0.0526316	**13**
0.00331251	0.00183709	0.0153846	0.04	**15**
0.00347222	0.00201936	0.0181818	0.0454545	**17**
0.00193124	0.00193124	0.0526316	0.0181818	**18**
0.00378788	0.00214163	0.0109898	0.0192308	**19**
0.00439721	0.00156871	0.0357143	0.0357143	**20**
0.00437691	0.00182815	0.030303	0.047619	**22**
0.0054319	0.00159236	0.0588235	0.0344828	**23**
0.00445207	0.00186567	0.0344828	0.025641	**24**

Applying model (6) and (5) for table (2) respectively we have $\varepsilon_1^+ = 0.0181818$, $\varepsilon_2^+ = 0.010989$ for inputs and $\varepsilon_1^+ = 0.00149332$, $\varepsilon_2^+ = 0.00193124$ for outputs.

We solve model (7) using the bounds obtained from (5) and (6) and the results will be shown in table III:

Table III. Efficiency of DMUs after modification

	DMU
0.832018	1
0.62444	2
0.626655	3
0.521402	4
0.80266	5
0.369739	6
0.381914	7
1	8
0.259239	9
1	10
0.993969	11
0.610471	12
0.336349	13
0.3529303	14
0.602154	15
0.835751	16
0.878793	17
0.625904	18
0.344945	19
0.659964	20
0.689795	21
0.955311	22
1	23
0.715147	24
0.73120	25

IV. CONCLUSION

In this article we modify the recyclable outputs via modified CCR model. In The basic DEA models, it is assumed that the output produced is go out of the system and considered as final outputs. In Amirteimoori model some parts of produced outputs used as inputs in the system again. The outputs that used in the system as inputs are called recyclable outputs. Note that the above procedure it is possible that the performances of some units are not properly diagnosed therefore, modifying the model by facet Analysis the exact decision making units' performance is achieved. Facet analysis gives information about hyper planes that located on weak boundaries. This information helps us to modify the boundaries so the exact efficiency and inefficiency of all DMUs especially DMUs which located on weak boundaries or DMUs that compared with weak parts of boundaries.

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