STOCHASTIC MODELLING OF A COMPUTER SYSTEM WITH HARDWARE REDUNDANCY SUBJECT TO MAXIMUM REPAIR TIME

V.J. Munday*

Department of Statistics, M.D. University, Rohtak-124001 (India)
Email: vjmunday@rediffmail.com

S.C. Malik

Department of Statistics, M.D. University, Rohtak-124001 (India)
Email: sc_malik@rediffmail.com

Permila

Department of Statistics, M.D. University, Rohtak-124001 (India)
Email: permilasingh88@gmail.com

Abstract

A computer system with hardware redundancy in cold standby has been analyzed stochastically by giving maximum repair time to the server. The components may fail independently from normal mode. A single server is immediately provided to carry out repair activities. The failed hardware is replaced by new one in case its repair is not feasible by the server in a given maximum repair time. However, software is up-graded when it fails to function as per requirements. The failure time of the components follow negative exponential distribution whereas the distributions of repair, up-gradation and replacement times are taken as arbitrary. The semi-Markov process and regenerative point technique are used to analyze the system model. The behaviour of some reliability measures have been observed graphically for arbitrary values of the parameters. The profit of the present model has also been compared with the model Malik and Munday (2014).

Keywords*:* Computer System; Hardware Redundancy; Up-gradation; Replacement; Maximum Repair Time; Profit Comparison; Stochastic Modelling.

1. Introduction

The expectations of the modern society on the use of systems of high performance have increased exponentially during past few years. The performance of systems can be improved by adopting proper repair policies and components of low failure rates. The technique of redundancy has also been considered as a tool to enhance durability of the systems. The scientists and engineers have proved that the technique of cold standby redundancy in computer systems is useful for improving their performance. It is a known fact that most of the academic and analytical works in institutions and offices have been performed by computer systems. Therefore, software and hardware firms are stressing on the development of faultless computer systems with latest technology. And, a major challenge to the manufacturers is also to provide reliable components with minimum overall costs. Most of the academicians are also trying to explore new techniques for reliability improvement of the computer systems. In spite of these efforts, a little work has been dedicated to the reliability modelling of computer systems. And, most of the research work carried out so far in the subject of hardware and software reliability has been limited to the consideration of either hardware or software subsystem alone. Osaki and Nishio (1979) and Lai et al. (2002) evaluated the availability analysis of distributed hardware/software systems. Malik and Anand (2010, 2011), Malik and Sureria (2012) and Kumar and Malik (2013) discussed reliability models of a computer system considering different repair policies. Recently, Malik and Munday (2014) studied a cold standby computer system with hardware repair and software up-gradation by a server who visits the system immediately whenever needed.

While considering the practical importance of computer systems in our day to day work, the stochastic modelling of a computer system has been done by providing hardware redundancy in cold standby and maximum repair time to the server. The system comprises hardware and software components which have independent failure via normal mode. A single server visits the system immediately as and when needed. The hardware component under goes for repair at its failure and replaced by new one in case it is not repaired up to a fixed repair time. However, software component is up-graded at its failure. The failure time distribution of the components is taken as negative exponential whereas the distributions of up-gradation time, repair time and replacement time are assumed as arbitrary with different probability density functions. Various performance measures of the system model such as transition probabilities, mean sojourn times, mean time to system failure (MTSF), availability, busy period of the server due to hardware repair and replacement, busy period of the server due to software up-gradation, expected number of hardware repairs, expected number of software up-gradations, expected number of hardware replacements by the server and profit function have been obtained using semi-Markov process and regenerative point technique. The graphical study has also been made to depict the behaviour of some important reliability measures. The profit comparison of the system model has also been made with that of Malik and Munday (2014) for different parametric values.

2. Notations

E	:	Set of regenerative states
$\bar{E}$	:	Set of non-regenerative states
O	:	Computer system is operative
Hcs	:	Hardware is in cold standby
a/b	:	Probability that the system has hardware / software failure
λ_1/ λ_2	:	Hardware/Software failure rate
α_0	:	Rate for which component undergoes for replacement after a maximum repair time t
HFUr /HFWr	:	The hardware is failed and under/waiting for repair
SFUg/SFWUg	:	The software is failed and under/waiting for up-gradation
HFUR/HFWR	:	The hardware is failed and continuously under/ waiting for repair from previous state
SFUG/SFWUG	:	The software is failed and continuously under /waiting for up- gradation from previous state
g(t)/G(t)	:	pdf/cdf of hardware repair time
f(t)/F(t)	:	pdf/cdf of software up-gradation time
r(t)/R(t)	:	pdf/cdf of hardware replacement time
$q_{ij}(t)$/ $Q_{ij}(t)$	:	pdf / cdf of first passage time from regenerative state S_i to a regenerative state S_j or to a failed state S_j without visiting any other regenerative state in (0, t]
$q_{ij.k}$ (t)/$Q_{ij.k}(t)$	:	pdf/cdf of direct transition time from regenerative state S_i to a regenerative state S_j or to a failed state S_j visiting state S_k once in (0, t]
$q_{ij.kl}$ (t)/$Q_{ij.kl}(t)$	:	pdf/cdf of direct transition time from regenerative state S_i to a regenerative state S_j or to a failed state S_j visiting state S_k and S_l once in (0, t]
$M_i(t)$	:	Probability that the system up initially in state $S_i \in$ E is up at time t without visiting to any regenerative state
$W_i(t)$	:	Probability that the server is busy in the state S_i up to time 't' without making any transition to any other regenerative state or returning to the same state via one or more non-regenerative states.
μ_i	:	The mean sojourn time in state S_i which is given by $\mu_i = E(T) = \int_0^\infty P(T > t)\, dt = \sum_j m_{ij}$, where T denotes the time to system failure.
m_{ij}	:	Contribution to mean sojourn time (μ_i) in state S_i when system transits directly to state S_j so that $\mu_i = \sum_j m_{ij}$ *and* $m_{ij} = \int_0^\infty t dQ_{ij}(t) = -q_{ij}^{*'}(0)$
Ⓢ/©	:	Symbol for Laplace-Stieltjes convolution/Laplace convolution
*/**	:	Symbol for Laplace Transformation (LT)/Laplace Stieltjes Transformation (LST)

State Transition Diagram

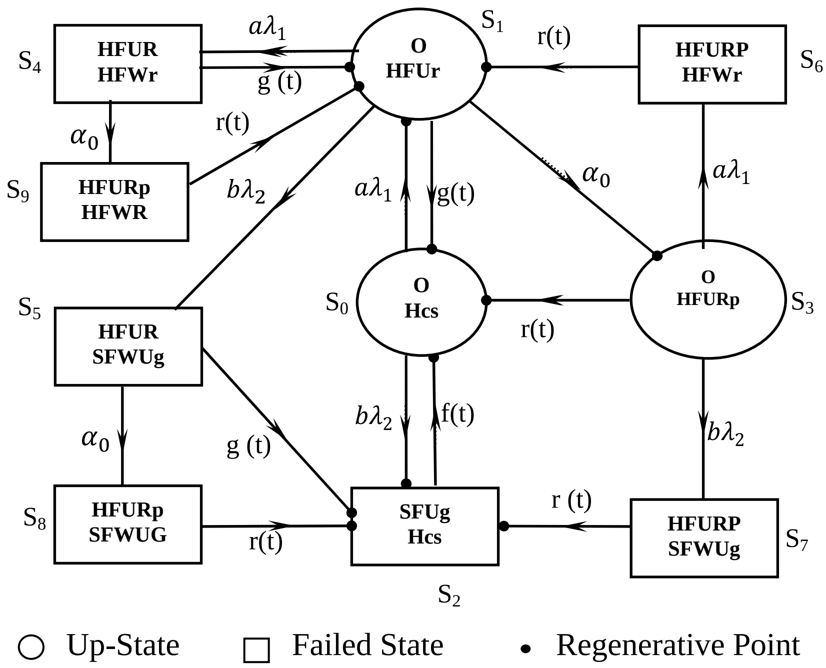

Fig. 1

3. Transition Probabilities and Mean Sojourn Times

Simple probabilistic considerations yield the following expressions for the non-zero elements.

$p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}\,(t)dt$

$p_{01} = \frac{a\lambda_1}{a\lambda_1+b\lambda_2}$, $p_{02} = \frac{b\lambda_2}{a\lambda_1+b\lambda_2}$, $p_{10} = g^*(a\lambda_1 + b\lambda_2 + \alpha_0)$, $p_{13} = \frac{\alpha_0}{a\lambda_1+b\lambda_2+\alpha_0}\{1 - g^*(a\lambda_1 + b\lambda_2 + \alpha_0)$

$p_{14} = \frac{a\lambda_1}{a\lambda_1+b\lambda_2+\alpha_0}\{1 - g^*(a\lambda_1 + b\lambda_2 + \alpha_0)$, $p_{15} = \frac{b\lambda_2}{a\lambda_1+b\lambda_2+\alpha_0}\{1 - g^*(a\lambda_1 + b\lambda_2 + \alpha_0)$

$p_{20} = f^*(0)$, $p_{30} = r^*(a\lambda_1 + b\lambda_2)$, $p_{36} = \frac{a\lambda_1}{a\lambda_1+b\lambda_2}\{1 - r^*(a\lambda_1 + b\lambda_2)$, $p_{37} = \frac{b\lambda_2}{a\lambda_1+b\lambda_2}\{1 - r^*(a\lambda_1 + b\lambda_2)$,
$p_{41} = p_{52} = g^*(\alpha_0)$, $p_{49} = p_{58} = 1 - g^*(\alpha_0)$, $p_{61} = p_{72} = p_{82} = p_{91} = r^*(0)$

For $g(t) = \alpha e^{-\alpha t}$, $f(t) = \theta e^{-\theta t}$ *and* $r(t) = \beta e^{-\beta t}$ we have

But, $f^*(0) = g^*(0) = r^*(0) = 1$ *and* $a + b = 1$

$p_{11.4} = \frac{\alpha a\lambda_1}{(\alpha+\alpha_0)(a\lambda_1+b\lambda_2+\alpha+\alpha_0)}$, $p_{11.49} = \frac{\alpha_0 a\lambda_1}{(\alpha+\alpha_0)(a\lambda_1+b\lambda_2+\alpha+\alpha_0)}$

$p_{12.5} = \frac{\alpha b\lambda_2}{(\alpha+\alpha_0)(a\lambda_1+b\lambda_2+\alpha+\alpha_0)}$, $p_{12.58} = \frac{\alpha_0 b\lambda_2}{(\alpha+\alpha_0)(a\lambda_1+b\lambda_2+\alpha+\alpha_0)}$

$p_{31.6} = \frac{a\lambda_1}{a\lambda_1+b\lambda_2+\beta}$, $p_{32.7} = \frac{b\lambda_2}{a\lambda_1+b\lambda_2+\beta}$

It can be easily verified that

$$p_{01} + p_{02} = p_{10} + p_{13} + p_{14} + p_{15} = p_{20} = p_{30} + p_{36} + p_{37} = p_{41} + p_{49} = p_{52} + p_{58} = p_{61}$$
$$= p_{72} = p_{82} = p_{91} = p_{10} + p_{13} + p_{11.4} + p_{11.49} + p_{12.5} + p_{12.58} = p_{30} + p_{31.6} + p_{32.7} = 1$$

The mean sojourn times (μ_i) is the state S_i are

$\mu_0 = \frac{1}{a\lambda_1+b\lambda_2}$ $\mu_1 = \frac{1}{a\lambda_1+b\lambda_2+\alpha_0+\alpha}$ $\mu_2 = \frac{1}{a\lambda_1+b\lambda_2+\beta}$

$\mu_1^{'} = \frac{(\alpha+\beta)(a\lambda_1+b\lambda_2)+\beta(\alpha+\alpha_0)}{\beta(\alpha+\alpha_0)(a\lambda_1+b\lambda_2+\alpha+\alpha_0)}$, $\mu_3^{'} = \frac{1}{\beta}$

Also $m_{01}+m_{02}=\mu_0,\ m_{10}+m_{13}+m_{14}+m_{15}=\mu_1,\ m_{30}+m_{36}+m_{37}=\mu_3$

And $m_{10}+m_{13}+m_{11.4}+m_{11.49}+m_{12.5}+m_{12.58}=\mu_1^{'}$, $m_{30}+m_{31.6}+m_{32.7}=\mu_3^{'}$

4. Reliability and Mean Time to System Failure (MTSF)

Let $\phi_i(t)$ be the cdf of first passage time from regenerative state $S_i$to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for $\phi_i(t)$,

$$\phi_0(t)=Q_{01}(t)\,Ⓢ\,\phi_1(t)+Q_{02}(t)$$

$$\phi_1(t)=Q_{10}(t)\,Ⓢ\,\phi_0(t)+Q_{13}(t)\,Ⓢ\,\phi_3(t)+Q_{14}(t)+Q_{15}(t)$$

$$\phi_3(t)=Q_{30}(t)\,Ⓢ\,\phi_0(t)+Q_{36}(t)+Q_{37}(t) \tag{1}$$

Taking LST of above relations (1) and solving for $\phi_0^{**}(s)$, we have

$$R^*(s)=\frac{1-\phi_0^{**}(s)}{s}$$

The reliability of the system model can be obtained by taking Laplace inverse transform of the above equation. The mean time to system failure (MTSF) is given by

$$MTSF=\lim_{s\to 0}\frac{1-\phi_0^{**}(s)}{s}=\frac{N_1}{D_1} \tag{2}$$

Where $N_1=\mu_0+p_{01}(\mu_1+p_{13}\mu_3)\ and\ D_1=1-p_{01}(p_{10}+p_{13}p_{30})$ (3)

5. Steady State Availability

Let $A_i(t)$ be the probability that the system is in up-state at an instant‘t’ given that the system entered regenerative state $S_i\ at\ t=0$. The recursive relations for $A_i(t)$ are given as:

$$A_0(t)=M_0(t)+q_{01}(t)©A_1(t)+q_{02}(t)©A_2(t)$$

$$A_1(t)=M_1(t)+q_{10}(t)©A_0(t)+\{q_{11.4}(t)+q_{11.49}(t)\}©A_1(t)+\{q_{12.5}(t)+q_{12.58}(t)\}©A_2(t)+q_{13}(t)©A_3(t)$$

$$A_2(t)=q_{20}(t)©A_0(t)$$

$$A_3(t)=M_3(t)+q_{30}(t)©A_0(t)+q_{31.6}(t)©A_1(t)+q_{32.7}(t)©A_2(t) \tag{4}$$

where

$$M_0(t)=e^{-(a\lambda_1+b\lambda_2)t},\ \mathrm{M}_1(t)=e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}\,\overline{G(t)}\ and\ \mathrm{M}_3(t)=e^{-(a\lambda_1+b\lambda_2)t}\,\overline{R(t)}$$

Taking LT of relations (4) and solving for $A_0^*(s)$, the steady state availability is given by

$$A_0(\infty)=\lim_{s\to 0}s\,A_0^*(s)=\frac{N_2}{D_2} \tag{5}$$

Where $N_2=\mu_0(1-p_{11.4}-p_{11.49}-p_{13}p_{31.6})+p_{01}(\mu_1+\mu_3p_{13})$

$$D_2=\mu_0(1-p_{11.4}-p_{11.49}-p_{13}p_{31.6})+p_{01}(\mu_1^{'}+\mu_3^{'}p_{13})+\mu_2(p_{12.5}+p_{12.58}+p_{13}p_{32.7}$$
$$+p_{02}(p_{10}+p_{13}p_{30})) \tag{6}$$

6. Busy Period of the Server

6.1 Due to Hardware Repair

Let $B_i^H(t)$ be the probability that the server is busy in repairing the unit due to hardware failure at an instant‘t’ given that the system entered state $S_i\ at\ t=0$. The recursive relations for $B_i^H(t)$ are as follows:

$$B_0^H(t)=q_{01}(t)©B_1^H(t)+q_{02}(t)©B_2^H(t)$$

$$B_1^H(t)=W_1^H(t)+q_{10}(t)©B_0^H(t)+\{q_{11.4}(t)+q_{11.49}(t)\}©B_1^H(t)+\{q_{12.5}(t)+q_{12.58}(t)\}©B_2^H(t)$$
$$+q_{13}(t)©B_3^H(t)$$

$$B_2^H(t)=q_{20}(t)©B_0^H(t)$$

$$B_3^H(t)=q_{30}(t)©B_0^H(t)+q_{31.6}(t)©B_1^H(t)+q_{32.7}(t)©B_2^H(t) \tag{7}$$

where $W_1^H(t) = e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}\,\overline{G(t)} + (a\lambda_1 e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}©1)\overline{G(t)} + (b\lambda_2 e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}©1)\overline{G(t)}$

6.2 Due to Software Up-gradation

Let $B_i^S(t)$ be the probability that the server is busy in upgrading the unit due to software failure at an instant't' given that the system entered state $S_i\ at\ t = 0$. The recursive relations for $B_i^S(t)$ are as follows:

$B_0^S(t) = q_{01}(t)©B_1^S(t) + q_{02}(t)©B_2^S(t)$

$B_1^S(t) = q_{10}(t)©B_0^S(t) + \{q_{11.4}(t) + q_{11.49}(t)\}©B_1^S(t) + \{q_{12.5}(t) + q_{12.58}(t)\} © B_2^S(t) + q_{13}(t)©B_3^S(t)$

$B_2^S(t) = W_2^S(t) + q_{20}(t)©B_0^S(t)$

$B_3^S(t) = q_{30}(t)©B_0^S(t) + q_{31.6}(t)©B_1^S(t) + q_{32.7}(t)©B_2^S(t)$ (8)

where $W_2^S(t) = \overline{F(t)}$

6.3 Due to Hardware Replacement

Let $B_i^{Rp}(t)$ be the probability that the server is busy in replacement of the unit due to hardware failure after an instant't' given that the system entered state $S_i\ at\ t = 0$. The recursive relations for $B_i^S(t)$ are as follows:

$B_0^{Rp}(t) = q_{01}(t)©B_1^{Rp}(t) + q_{02}(t)©B_2^{Rp}(t)$

$$B_1^{Rp}(t) = q_{10}(t)©B_0^{Rp}(t) + \{q_{11.4}(t) + q_{11.49}(t)\}©B_1^{Rp}(t) + \{q_{12.5}(t) + q_{12.58}(t)\} © B_2^{Rp}(t) + q_{13}(t)©B_3^{Rp}(t)$$

$B_2^{Rp}(t) = q_{20}(t)©B_0^{Rp}(t)$

$B_3^{Rp}(t) = W_3^{Rp}(t) + q_{30}(t)©B_0^{Rp}(t) + q_{31.6}(t)©B_1^{Rp}(t) + q_{32.7}(t)©B_2^{Rp}(t)$ (9)

where $W_3^{Rp}(t) = e^{-(a\lambda_1+b\lambda_2)t}\,\overline{R(t)} + (a\lambda_1 e^{-(a\lambda_1+b\lambda_2)t})\overline{R(t)} + (b\lambda_1 e^{-(a\lambda_1+b\lambda_2)t})\overline{R(t)}$

Taking LT of relations (7), (8) & (9), solving for $B_0^{H^*}(t), B_0^{S^*}(t)\ and\ B_0^{Rp^*}(t)$. The time for which server is busy due to repairs, up-gradations and replacements respectively are given by

$B_0^H(t) = \lim_{s\to 0} s\,B_0^{H^*}(t) = \frac{N_3^H}{D_2}$ (10)

$B_0^S(t) = \lim_{s\to 0} s\,B_0^{S^*}(t) = \frac{N_3^S}{D_2}$ (11)

$B_0^{Rp}(t) = \lim_{s\to 0} s\,B_0^{Rp^*}(t) = \frac{N_3^{Rp}}{D_2}$ (12)

where $N_3^H = p_{01}W_1^{H^*}(0)$

$N_3^S = ((p_{12.5} + p_{12.58}) + p_{10}p_{02} + p_{13}p_{32.7} + p_{13}p_{02}(p_{30} - 1))W_2^{S^*}$

$N_3^{Rp} = p_{01}p_{13}W_3^{Rp^*}(0)\ and\ D_2\ is\ already\ mentioned.$ (13)

7. Expected Number of Hardware Repairs

Let $NHR_i(t)$ be the expected number of hardware repairs by the server in (0, t] given that the system entered the regenerative state $S_i\ at\ t = 0$. The recursive relations for $NHR_i(t)$ are given as:

$NHR_0(t) = Q_{01}(t)Ⓢ NHR_1(t) + Q_{02}(t)Ⓢ NHR_2(t)$

$$NHR_1(t) = Q_{10}(t)Ⓢ(1 + NHR_0(t)) + Q_{11.4}(t)Ⓢ(1 + NHR_1(t) + Q_{11.49}(t)Ⓢ NHR_1(t) + Q_{12.5}(t)Ⓢ(1 + NHR_2(t)) + Q_{12.58}(t)Ⓢ NHR_2(t) + Q_{13}(t)Ⓢ NHR_3(t)$$

$NHR_2(t) = Q_{20}(t)Ⓢ NHR_0(t)$

$NHR_3(t) = Q_{30}(t)Ⓢ NHR_0(t) + Q_{31.6}(t)Ⓢ NHR_1(t) + Q_{32.7}(t)Ⓢ NHR_2(t)$ (14)

Taking LST of relations (14) and solving for $NHR_0^{**}(s)$. The expected number of hardware repair is given by

$NHR_0 = \lim_{s \to 0} sNHR_0^{**}(s) = \frac{N_4}{D_2}$ (15)

where $N_4 = p_{01}(p_{10} + p_{11.4} + p_{12.5})$ *and* D_2 *is already mentioned.* (16)

8. Expected Number of Software Up-gradations

Let $NSU_i(t)$ be the expected number of software up-gradations in (0, t] given that the system entered the regenerative state S_i *at* $t = 0$. The recursive relations for $NSU_i(t)$ are given as follows:

$$NSU_0(t) = Q_{01}(t) ⓢ NSU_1(t) + Q_{02}(t) ⓢ NSU_2(t)$$

$$NSU_1(t) = Q_{10}(t) ⓢ NSU_0(t) + \{Q_{11.4}(t) + Q_{11.49}(t)\} ⓢ NSU_1(t) + \{Q_{12.5}(t) + Q_{12.58}(t)\} ⓢ NSU_2(t)$$
$$+Q_{13}(t) ⓢ NSU_3(t)$$

$$NSU_2(t) = Q_{20}(t) ⓢ (1 + NSU_0(t))$$

$$NSU_3(t) = Q_{30}(t) ⓢ NSU_0(t) + Q_{31.6}(t) ⓢ NSU_1(t) + Q_{32.7}(t) ⓢ NSU_2(t) \quad (17)$$

Taking LST of relations (17) and solving for $NSU_0^{**}(s)$. The expected numbers of software up-gradations are given by

$NSU_0(\infty) = \lim_{s \to 0} sNSU_0^{**}(s) = \frac{N_5}{D_2}$ (18)

Where

$N_5 = p_{12.5} + p_{12.58} + p_{10}p_{02} + p_{13}p_{32.7} + p_{13}p_{02}(p_{30} - 1)$

and D_2 *is already mentioned.* (19)

9. Expected Number of Hardware Replacements

Let $NHRp_i(t)$ be the expected number of hardware replacements by the server in (0, t] given that the system entered the regenerative state S_i *at* $t = 0$. The recursive relations for $NHRp_i(t)$ are given as:

$$NHRp_0(t) = Q_{01}(t) ⓢ NHRp_1(t) + Q_{02}(t) ⓢ NHRp_2(t)$$

$$NHRp_1(t) = Q_{10}(t) ⓢ NHRp_0(t) + Q_{11.4}(t) ⓢ NHRp_1(t) + Q_{11.49}(t) ⓢ (1 + NHRp_1(t))$$
$$+Q_{12.5}(t) ⓢ NHRp_2(t) + Q_{12.58}(t) ⓢ (1 + NHRp_2(t)) + Q_{13}(t) ⓢ NHRp_3(t)$$

$$NHRp_2(t) = Q_{20}(t) ⓢ NHRp_0(t)$$

$$NHRp_3(t) = Q_{30}(t) ⓢ \left(1 + NHRp_0(t)\right) + Q_{31.6}(t) ⓢ \left(1 + NHRp_1(t)\right) + Q_{32.7}(t) ⓢ \left(1 + NHRp_2(t)\right) \quad (20)$$

Taking LST of relations (20) and solving for $NHRp_0^{**}(s)$. The expected numbers of hardware replacements are given by

$NHR_0 = \lim_{s \to 0} sNHRp_0^{**}(s) = \frac{N_6}{D_2}$ (21)

where $N_6 = p_{01}(p_{13} + p_{11.49} + p_{12.58})$ *and* D_2 *is already mentioned.* (22)

10. Profit Analysis

The profit incurred to the system model in steady state can be obtained as:

$P = K_0A_0 - K_1B_0^H - K_2B_0^S - K_3NHR_0 - K_4NSU_0 - K_5B_0^{Rp} - K_6NHRp_0$ (23)

where

$K_0 =$ Revenue per unit up-time of the system

$K_1 =$ Cost per unit time for which server is busy due to hardware repair

$K_2 =$ Cost per unit time for which server is busy due to software up-gradation

$K_3 =$ Cost per unit repair of the failed hardware

$K_4 =$ Cost per unit up-gradation of the failed software

$K_5 =$ Cost per unit time for which server is busy due to hardware replacement

$K_6 =$ Cost per unit replacement of the failed hardware

and $A_0, B_0^H, B_0^S, NHR_0, NSU_0, B_0^{Rp}, NHRp_0$ are already defined.

11. Particular Cases

Suppose $g(t) = \alpha e^{-\alpha t}$, $f(t) = \theta e^{-\theta t}$ and $r(t) = \beta e^{-\beta t}$

We can obtain the following results:

$$MTSF\ (T_0) = \frac{N_1}{D_1}$$

$$Availability\ (A_0) = \frac{N_2}{D_2}$$

Busy period due to hardware failure $(B_0^H) = \dfrac{N_3^H}{D_2}$

Busy period due to software failure $(B_0^S) = \dfrac{N_3^S}{D_2}$

Busy period due to replacement of hardware failure $\left(B_0^{Rp}\right) = \dfrac{N_3^{Rp}}{D_2}$

Expected number of repair at hardware failure $(NHR_0) = \dfrac{N_4}{D_2}$

Expected number of up − gradation at software failure $(NSU_0) = \dfrac{N_5}{D_2}$

Expected number of replacement at hardware failure $(NHRp_0) = \dfrac{N_6}{D_2}$

where

$$N_1 = \frac{(a\lambda_1 + b\lambda_2 + \alpha + \alpha_0)(a\lambda_1 + b\lambda_2 + \beta) + a\lambda_1(a\lambda_1 + b\lambda_2 + \beta + \alpha_0)}{(a\lambda_1 + b\lambda_2)(a\lambda_1 + b\lambda_2 + \alpha + \alpha_0)(a\lambda_1 + b\lambda_2 + \beta)}$$

$$D_1 = \frac{(a\lambda_1 + b\lambda_2 + \alpha + \alpha_0)(a\lambda_1 + b\lambda_2 + \beta) - a\lambda_1(\alpha(a\lambda_1 + b\lambda_2 + \beta) + \alpha_0\beta)}{(a\lambda_1 + b\lambda_2)(a\lambda_1 + b\lambda_2 + \alpha + \alpha_0)(a\lambda_1 + b\lambda_2 + \beta)}$$

$$N_2 = \frac{1}{(a\lambda_1 + b\lambda_2)}$$

$$D_2 = \frac{\begin{array}{c}\beta(\alpha+\alpha_0)(a\lambda_1+b\lambda_2+\alpha+\alpha_0)(\theta((a\lambda_1+b\lambda_2+\beta)(a\lambda_1+b\lambda_2+\alpha+\alpha_0)-a\lambda_1(a\lambda_1+b\lambda_2+\beta+\alpha_0))+b\lambda_2((a\lambda_1+b\lambda_2+\beta+\alpha_0) \\ (a\lambda_1+b\lambda_2)+\alpha(a\lambda_1+b\lambda_2+\beta)+\alpha_0\beta))+a\lambda_1\theta(a\lambda_1+b\lambda_2+\beta)(\beta(\alpha+\alpha_0)^2+(a\lambda_1+b\lambda_2)(\beta(a\lambda_1+b\lambda_2+2(\alpha+\alpha_0)+ \\ \alpha(a\lambda_1+b\lambda_2+\alpha+\alpha_0))+\beta\alpha_0(\alpha+\alpha_0)(a\lambda_1+b\lambda_2+\alpha+\alpha_0)\end{array}}{\beta\theta(a\lambda_1+b\lambda_2)(a\lambda_1+b\lambda_2+\alpha+\alpha_0)^2(a\lambda_1+b\lambda_2+\beta)(\alpha+\alpha_0)}$$

$$N_3^H = \frac{a\lambda_1}{(a\lambda_1+b\lambda_2)(a\lambda_1+b\lambda_2+\beta)}, \qquad N_3^S = \frac{b\lambda_2(a\lambda_1+b\lambda_2+\alpha)}{\theta(a\lambda_1+b\lambda_2)(a\lambda_1+b\lambda_2+\alpha+\alpha_0)}$$

$$N_3^{Rp} = \frac{a_0 a\lambda_1}{(a\lambda_1+b\lambda_2)(a\lambda_1+b\lambda_2+a+\alpha_0)(a\lambda_1+b\lambda_2+\beta)}, \qquad N_4 = \frac{\alpha a\lambda_1}{(a\lambda_1+b\lambda_2)(\alpha+\alpha_0)}$$

$$N_5 = \frac{b\lambda_2(a\lambda_1+b\lambda_2+\alpha)}{(a\lambda_1+b\lambda_2)(a\lambda_1+b\lambda_2+\alpha+\alpha_0)}, \qquad N_6 = \frac{\alpha_0 a\lambda_1}{(a\lambda_1+b\lambda_2)(\alpha+\alpha_0)}$$

12. Conclusion

The effect of various parameters on the reliability measures of a computer system has been observed for $g(t) = \alpha e^{-\alpha t}$, $f(t) = \theta e^{-\theta t}$ and $r(t) = \beta e^{-\beta t}$ as shown in figures 2, 3, and 4. It is analyzed that mean time to system failure (MTSF), availability and profit function go on decreasing with the increase of failure rates (λ_1 and λ_2) while their values increase with the increase of hardware repair rate (α) and software up-gradation rate (θ) provided a > b. However, the effect software failure rate is more on these measures. On the other hand, system model will have less values of MTSF and availability in case of a > b but profit has more value. Hence, a computer system in which software has more chances of failure can be made more profitable by providing hardware redundancy in cold standby and maximum repair time to the server.

Comparative Study

The study reveals that present model is profitable over the model Malik and Munday (2014). And, so we analyze that a computer system can be made more profitable to use by giving maximum repair time to the server for getting repair of the failed hardware. Thus, the graphical behavior of the profit difference of the models with respect to hardware failure rate (λ_1) has been shown in figure 5.

Acknowledgement

The authors are grateful to the University Grants Commission (UGC), New Delhi, India for providing financial Assistance to carry out this research work under Rajiv Gandhi National Fellowship for SC Candidate.

References

[1] Anand, J.; Malik, S.C. (2011): Reliability Modelling of a Computer System with Priority for Replacement at Software Failure over Repair Activities at Hardware Failure. International Journal of Statistics and Systems, **6 (3)**, pp.315-325.

[2] Anand, Jyoti; Malik, S.C. (2012): Analysis of a Computer System with Arbitrary Distributions for H/W and S/W Replacement Time and Priority to Repair Activities of H/W over Replacement of the S/W. International Journal of Systems Assurance Engineering and Management, **3 (3)**, pp. 230-236.

[3] Kumar, A.; Malik, S.C. (2013): Reliability Modeling of a Computer System with Priority to PM over H/W Replacement Subject to MOT and MRT. Journal of Rajasthan Academy of Physical Sciences, **12 (2)**, pp. 199-212.

[4] Lai, C.D.; Xie, M.; Poh, K.L.; Dai, Y.S.; Yang, P. (2002): A Model for Availability Analysis of Distributed Software/Hardware Systems. Information and Software Technology, **44**, pp. 343-350.

[5] Malik, S.C.; Munday, V.J. (2014): Stochastic Modeling of a Computer System with Hardware Redundancy. International Journal of Computer applications, **89 (7)**, pp. 26-30.

[6] Osaki, Shunji; Nishio, Toshihiko (1979): Availability Evaluation of Redundant Computer System. Computers and Operations Research, **6**, pp. 87-97.

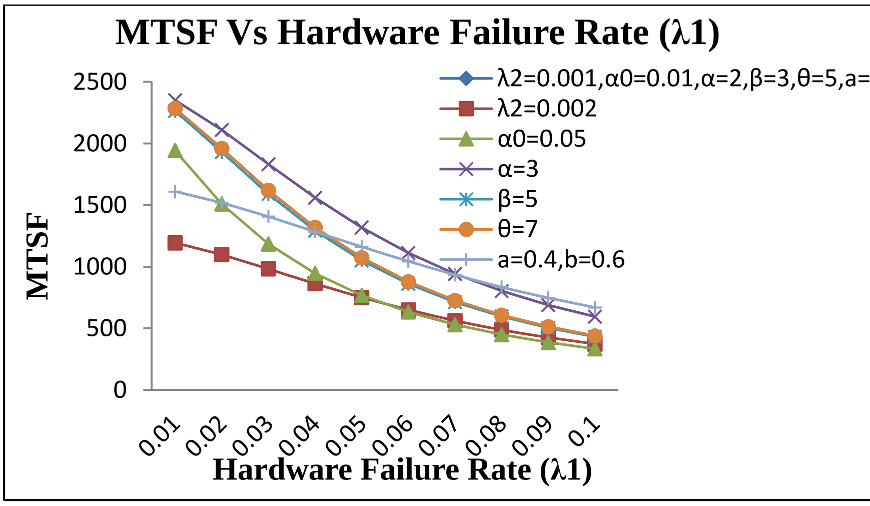

Fig. 2

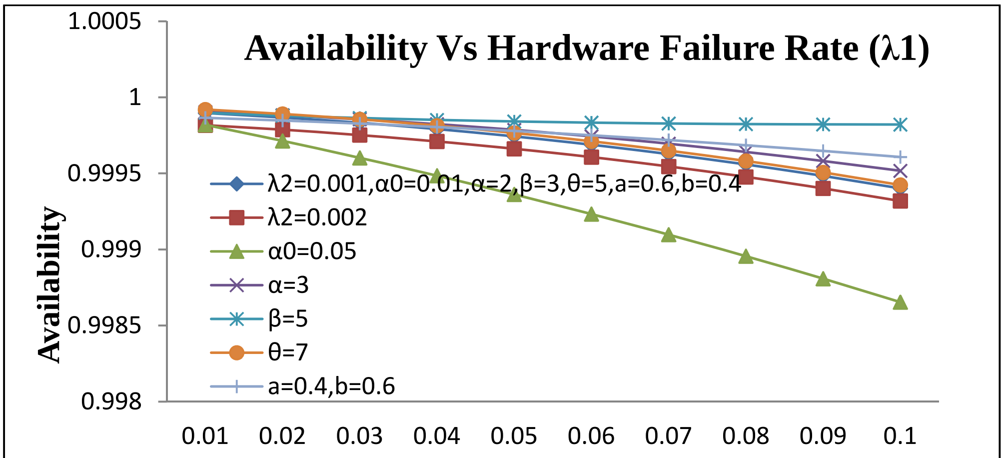

Fig. 3

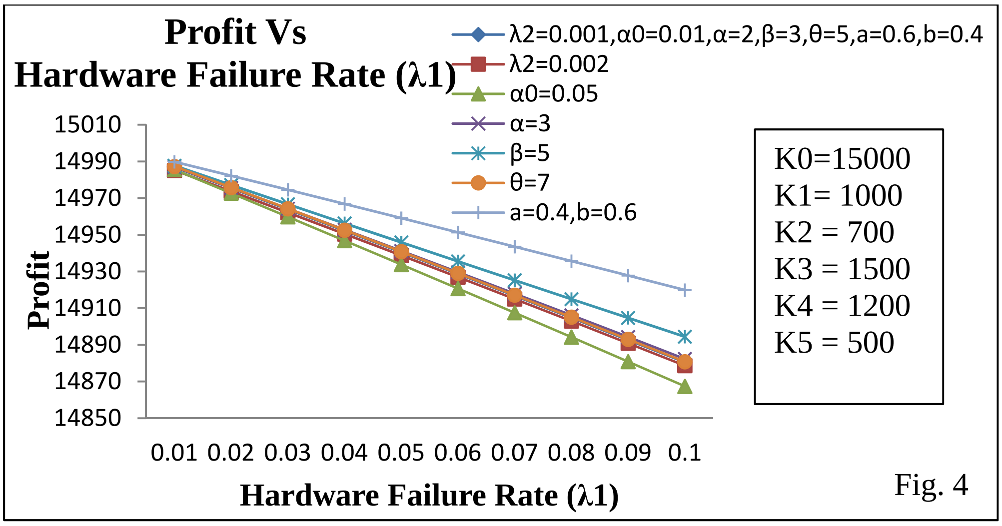

Fig. 4

P – P1 (Profit of Present Model – Profit of the Model Malik and Munday (2014))

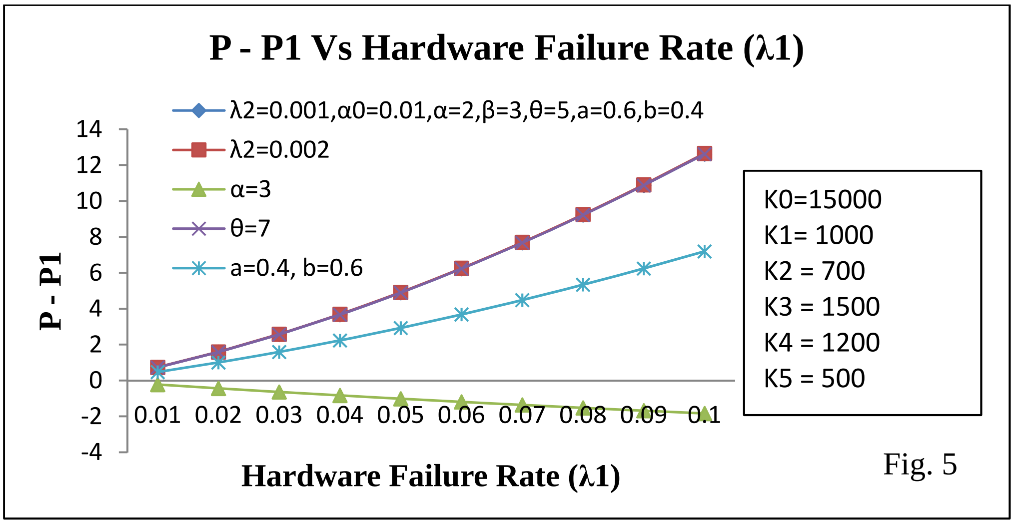

Fig. 5