Equivalence via Stochastic and Continuous Petri Nets for Modeling of Large Systems with Availability Constraints

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Abstract- High availability constraints ensure a high level of operational performance that must be maintained during a contractual measurement period. Such constraints must be satisfied, particularly, for critical systems related to global security and human safety. High availability often results from material redundancies and lead to large stochastic discrete event models with numerous states. Markov processes and stochastic Petri nets can be used to model, simulate and analyse such systems but their use is limited by the so called "combinatorial explosion" problem. This paper investigates the fluidification of stochastic Petri nets to overcome the previous problem. The main contribution is to propose a modular modeling of active and passive redundancies with Petri nets. Some approaches are then proposed to obtain equivalent behaviors in the long time with stochastic and continuous Petri nets.

General terms: SPN: stochastic petri net, PDF: probability density functions, contPNs: continuous Petri nets, rv: random variable

Keywords- Reliability analysis; Redundancies; Petri nets; Fluidification.

I. INTRODUCTION

Reliability and availability analysis are major challenges to improve the global security and the safety of processes. Users want their systems, for example, airplanes or computers, to be ready to serve them at all times. Availability refers to the ability of the user community to access the system, submit new work, update or alter existing work, or collect the results of previous work. If a user cannot access the system, it is said to be unavailable. The termsdowntime and uptimeare used to refer to periods when a system is unavailable and available. Scheduled and unscheduled downtime must be distinguished. Scheduled downtime results from maintenance operations and has little impact upon the user community. Unscheduled downtime events typically arise from hardware or software failures or environmental anomalies. Examples of unscheduled downtime events include power outages, failed CPU, over-temperature related shutdown, and so on. Such downtime must be considering at first in order to evaluate the availability of the system.

Mean availability can be expressed as the percentage of uptime in a given year. The table I show the downtime that will be allowed for a particular percentage of availability, presuming that the system is required to operate continuously (without maintenance periods).

Table I: Availability and downtime

Mean availability	Downtime per year
90% ("one nine")	36.5 days
95%	18.3 days
99% ("two nines")	3.7 days
99.5	1.8 days
99.9 ("three nines")	8.8 hours
99.99 ("four nines")	53 minutes
99.999 ("five nines")	5.3 minutes
99.9999 ("six nines")	32 seconds

High availability implies no human intervention to restore operation in complex systems. For example, availability limit of 99.999% allows about one second of downtime per day, which is impractical using human labor. Human intervention for maintenance actions will certainly exceed this limit. Availability limit of 99% would allow an average of 15 minutes per day, which is realistic for human intervention. So, high availability refers to availability at less equal to 99%.

Redundancy is used to eliminate the need for human intervention and to reach high availability requirements. For complex dynamical systems with numerous interdependent components and high availability constraints, the modeling and analysis methods are mainly based on stochastic discrete event models like Markov models [1] or stochastic Petri nets (SPNs) [2]. Such models are mathematically well founded and lead either to analytical results or numerical simulations. The first contribution of this paper is to propose modular models of the redundancies with SPNs. The compact and systematic design of SPNs is highlighted as an advantage in comparison with other state space models like Markov models. Estimation of availability with SPN simulations results as a consequence. But, in case of large systems, simulation and analysis methods lead to the problem of combinatory explosion that limits the use of state space models. In this context, fluidification can be discussed as a relaxation method [3,4]. The main idea of PNs fluidification is to replace a discrete SPN by a continuous one. The second contribution of the paper is to investigate some fluidification methods to evaluate the availability.

II. STOCHASTIC PETRI NETS FOR AVAILABILITY EVALUATION

A. Petri nets

A Petri net (PN) is defined as <***P***, ***T***, W_{PR}, W_{PO}> where ***P*** = {P_i} is a set of n places and ***T*** = {T_j} is a set of q transitions, $W = W_{PO} - W_{PR} \in (\mathbf{Z})^{n\times q}$ is the incidence matrix. $M(t, P_i)$ stands for the marking of place P_i at time t, $M(t)$ is the PN marking vector at time t and M_I the PN initial marking. The marking changes when a transition fires. Transitions fire depending on the marking vector and on the firing conditions. The marking variation is given with respect to the firing vector $X\in (\mathbf{Z})^q$ such that $\Delta M = W. X$. For a complete description of PN models and analysis, one can refer to [5,3].

B. Stochastic Petri nets

A stochastic Petri net (SPN) is a timed PN with transitions firing periods that are characterized by exponential probability density functions (pdf) with firing rate μ_j, $j = 1,\ldots,q$[6,2]. The marking of the place P_i of a marked SPN at time t will be referred as $M(t, P_i)$. The SPNs considered in this paper are bounded, reinitialisable, with infinite server semantic, race policy and resampling memory. As a consequence, the considered SPNs have a reachability graph with a finite number N of states {$S_1,\ldots,S_N$} and their marking process is mapped into a Markov model with state space isomorphic to the reachability graph [7]. The generator G of this Markov model can be computed from the reachability graph and from the firing parameters of the SPN. The state probabilities $\pi_k(t)$, $k = 1,\ldots,N$of Markov model and the mean marking $M_{mm}(t, Pi)$,$i = 1,\ldots,n$of SPN satisfy:

$$M_{mm}(t,Pi) = \sum_{k=1,\ldots,N} m_{k\,i}.\pi_k(t) \qquad (1)$$

wherem_{ki} stands for the number of tokens in place P_i when the system is in state S_k.

C. Example

Petri nets can be used to model systems with failures and repairs [8]. The simplest example is to consider a system composed of a single component (Figure 1). This component is assumed to fails and repairs with respect to exponential pdf. The failure rate is denoted by λ and μ stands for the repair rate. The place P_1 stands for the safe state and P_2 for the defect one. The transition T_1 represents the occurrence of a fault and the transition T_2 represents the end of the repair process.

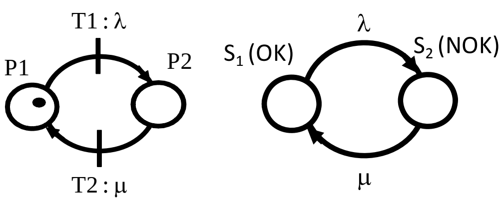

Fig 1: Failure and repair process of a single component; SPN (left); corresponding Markov model (right)

One can notice the similarity of both representations. In fact, the SPN in Figure 1 (left) is equivalent to the Markov model (right) with generator G:

$$G = \begin{pmatrix} -\lambda & \lambda \\ \mu & -\mu \end{pmatrix} \qquad (2)$$

The resolution of the Chapman Kolmogorov equation [1] leads to the determination of the mean availability $A(\infty)$ with respect to parameters λ and μ.

$$A(\infty) = \pi_1(\infty) = M_{mm}(\infty, P_1) = \frac{\mu}{\lambda + \mu} \tag{3}$$

To conclude with this example, one can notice that high availability requirements are achieved only if the repair process is at least 99 times quicker than the failure process.

D. Modular models of redundancies with PNs

The simple example provided with Figure 1 does not enhance the advantages to use SPNs in comparison with Markov models. Such advantages will appear as evidence when redundancies are considered. Two kinds of redundancies can be considered: passive redundancies and active redundancies.

E. Models of active redundant systems

Active redundancies are used to achieve high availability by including enough excess capacity in the design to accommodate a performance decline. Systems with active redundancies include several identical components working together. A simple example is an aircraft with two separate engines. The aircraft continues to fly despite failure of a single engine. A more complex example is multiple redundant power generation facilities within a large system involving electric power transmission. Malfunction of a single component is not considered to be a failure unless the resulting performance decline exceeds the specification limits for the entire system.

Figure 2 provides the usual representation of active redundancies with SPNs. A number n of identical component is considered. Components running simultaneously are represented by the tokens in place P_1. Non reparable (left) and reparable (right) processes are considered. In both cases, the sojourn time of any token in place P_1 is a random variable (rv) with exponential pdf of parameter λ.The duration to fire transition T_1 is also a rv with exponential pdf of parameter $\lambda.M(t, P_1)$. When a component fails or is repaired, this parameter changes as $M(t, P_1)$.

The SPNs in Figure 2 are used to represent total or partial active redundant system. A total redundant system is said to be available as long as $M(t, P_1) \geq 1$. In comparison a partial k / n redundant system is said to be available as long as $M(t, P_1) \geq k$. Mean availability with partial and total redundancies is easy to work out, with the evaluation of the marking $M(t, P_1)$.

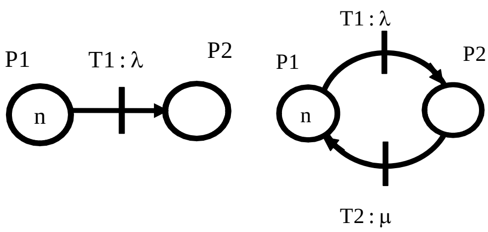

Fig 2: Active redundancies with n identical components; Failure process for non reparable systems (left); Failure and repair processes for reparable systems (right)

In Figure 3, the case of non identical components is considered. This representation will be preferred when the redundant components have different failure and repair rates. More precisely, in Figure 3, p classes of components are considered and n_i stands for the number of identical components in class i. In comparison with the previous model, the SPN in Figure 3 is composed of 2.p places, and availability will be evaluated according to the sum of marking variables $M(t, P_{11}) + \ldots + M(t, P_{p1})$.

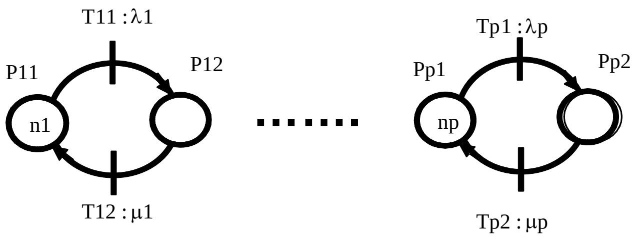

Fig 3: Failure and repair processes for active redundancies with p classes of ni components i = 1,…,p

F. Models of passive redundant systems

Passive redundancy is used in complex systems to achieve high availability with no performance decline. Multiple components are incorporated into a design that includes also a method to detect failures and automatically reconfigure the system to bypass failed items and replace them with safe ones. This is used with complex systems that are linked. For example, the rescue wind turbine in aircraft is a passive redundant component that will be used only when the power generators are out of order and when all batteries are down. In that case the rescue turbine will produce the energy required to maintain the main control devices of the aircraft.

Figure 4 provides the usual representation of passive redundancies with SPNs for reparable systems. The duration to fire transition T_1 is a rv with exponential pdf of parameter $\lambda.\min(M(t, P_1), M(t, P_3)) = \lambda.\min(M(t, P_1), 1)$. In Figure 4-left, concurrent repairs are considered and in Figure 4-right, non concurrent repairs are considered.

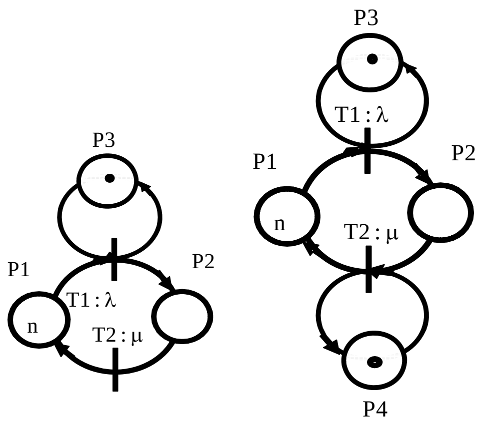

Fig 4: Failure and repair processes for passive redundancies; concurrent repairs (left); non concurrent repairs (right)

III. Availability evaluation with SPNs

A. *Analytical background*

A. SPNs can be used to evaluate usual indicators of reliability as characteristic times (MUT, MDT, MTTF or MTBF) and also instantaneous indicators as reliability or availability. In this work we will consider the particular case of mean availability. The basic idea is to use the equivalence that exists between the reachability graph of the SPN and the corresponding Markov model. The N states of the reachability graph are first separated into two classes: the class OK of N_s safe states and the class NOK of N_d defect ones. Let us denote $\Pi_s \in [0, 1]^{1 \times Ns}$ as the row vector of the state probabilities for normal states and $\Pi_d \in [0, 1]^{1 \times Nd}$ as the row vector of the state probabilities for defect ones. The Chapman Kolmogorov equation of the associated Markov model can be written as in (4):

$$\frac{d\left(\Pi_{OK}(t) \quad \Pi_{NOK}(t)\right)}{dt} = \left(\Pi_{OK}(t) \quad \Pi_{NOK}(t)\right).\begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix} \tag{4}$$

where G_{11}, G_{12}, G_{21} and G_{22} are sub-matrices of the generator G with appropriate dimensions. Thus, state probabilities are given by (5):

$$\Pi(t) = \Pi(0).exp\left(\begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix}.t\right) \tag{5}$$

and the mean availability is given by (6):

$$A(\infty) = \Pi(\infty).\begin{pmatrix} 1_{Ns} \\ 0_{Nd} \end{pmatrix} = \Pi_{OK}(\infty).1_{Ns} \tag{6}$$

with $1_{Ns} = (1,\dots 1)^T$ of dimension N_s.

B. *Stochastic estimator by means of simulations*

B. SPNs can be used to estimate the mean availability (and also other indicators) by means of simulations with time horizon D. For that purpose, the probability of each state S_k is estimated with equation (7):

$$\tilde{\pi}_k(D) = \frac{1}{D}\int_0^D f_k(t).dt \text{ and } \tilde{\pi}_k(\infty) = \lim_{D\to\infty}\left(\tilde{\pi}_k(D)\right) \tag{7}$$

The functions $f_k(t), k = 1,\dots,N$ are defined with equation (8):

$$f_k(t) = 1 \text{ if } M(t) = S_k \text{ and } f_k(t) = 0 \text{ if } M(t) \neq S_k \tag{8}$$

C. Examples:

C. Let us consider again the example of Figure 1 with parameters λ = 1.5e-5 TU^{-1} and μ = 1e-4 TU^{-1}. The simulation of the system over time interval [0 : 1e7] leads to the estimation of state S_1 probability reported in Figure 6. For this example, the availability equals the probability of state S_1 and tends to the mean availability A(∞) = 0.87. This numerical estimation coincides with the theoretical value of the mean availability provided by equation (2) and the considered system is not high available.

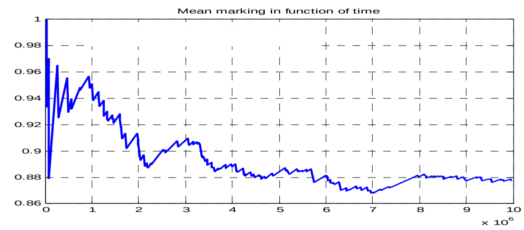

Fig 5: Estimation of the state S1 probability with SPN simulations for the system of Figure 1

Let us consider an active redundant system with 3 identical components with the same parameters λ = 1.5e-5 TU^{-1} and μ = 1e-4 TU^{-1}. The SPN and Markov model are described in Figure 6. The simulation of the SPN leads to the estimation of the mean availability *A(∞)* = 0.998 (Figure 7).

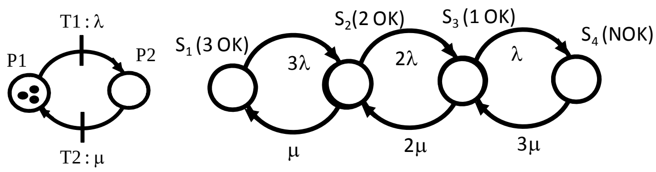

Fig6: Active redundancies with 3 identical components

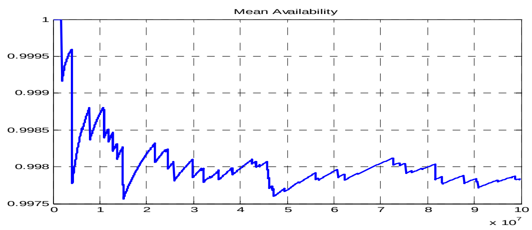

Fig 7: Estimation of the mean availability with SPN simulations for the system of Figure 6

D. PERFORMANCE EVALUATION FOR SYSTEMS WITH REDUNDANCIES

- *MODULAR MODELING*

As long as high availability is considered, systems with numerous redundant components must be represented. Each process proposed in figures 2 to 4 is a modular sub-model that can be included in the representation of a large system with redundancies. Figure 8 represents the connection of the failure and repair processes in large systems. The server with redundancies is represented by the transition T_3 that is connected to the sub-model {P_1, P_2, T_1, T_3} (a failure process for *n* reparable components with total active redundancies). A simplified representation of the system is used with {P_3, P_4, T_3, T_4}.

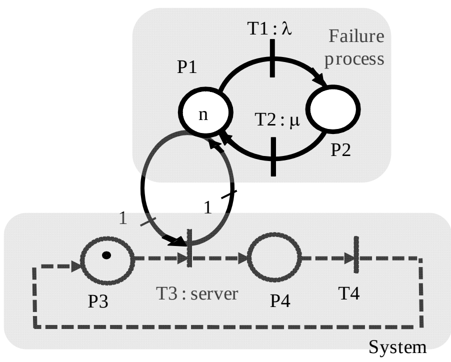

Fig 8: Failure and repair processes for active redundancies: integration of sub-models

The mean availability worked out with the sub-model in Figure 2 right and with the complete model in Figure 8 are equal. The proof is given for $n = 1$ and can be extended for $n > 1$. Assuming that the server T_3has a firing rate x and that T_4 has a firing rate y, the SPN in Figure 8 is equivalent to a Markov model with states $S_1 = (1\ 0\ 1\ 0)^T$, $S_2 = (0\ 1\ 1\ 0)^T$, $S_3 = (1\ 0\ 0\ 1)^T$, $S_4 = (0\ 1\ 0\ 1)^T$ and generator G:

$$G = \begin{pmatrix} -(\lambda + x) & \lambda & x & 0 \\ \mu & -\mu & 0 & 0 \\ y & 0 & -(y+\lambda) & \lambda \\ 0 & y & \mu & -(y+\mu) \end{pmatrix} \tag{9}$$

The mean availability is given by the asymptotic probabilities of states S_1 and S_3:

$$A(\infty) = \pi_1(\infty) + \pi_3(\infty) = \frac{1}{D}.\left(\frac{y}{x}.\left(\frac{\lambda+\mu+y}{\lambda}\right) + \frac{\mu+y}{\lambda}\right) \tag{10}$$

with:

$$D = \frac{y}{x}.\left(\frac{\lambda+\mu+y}{\lambda}\right) + \frac{y}{x}.\left(\frac{\lambda+\mu+x+y}{\mu}\right) + \frac{\mu+y}{\lambda} + 1$$

After simplification (3) may be rewritten with (11):

$$A(\infty) = \frac{1}{1 + \dfrac{\dfrac{y}{\mu}.(y+x+\mu+\lambda)+x}{\dfrac{y}{\lambda}.(y+\mu+\lambda)+\dfrac{x}{\lambda}.(y+\mu)}} = \frac{\mu+\lambda}{\lambda} \tag{11}$$

Thus the mean availability of SPNs in Figures 2 and 8 are identical and one can conclude that the integration of the sub-model for failure and repair process does not change the availability. The advantage of including the failure and repair processes in the global system is to evaluate the influence of availability on the server activity. In Figure 8, the flow of transition T_3 depends on the redundancies that are included in the system design. The previous proof can be generalized for n redundant components and for partial k / n redundancies (arcs (P_1, T_3) and (T_3, P_1) will be weighted with k).

- *Example*

The system in Figure 9models a simple manufacturing system.

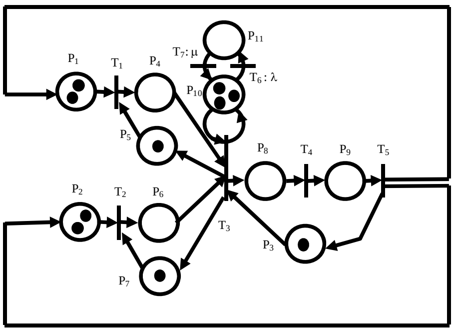

Figure 9: Assembly workshop

The final product is composed of two different parts, A and B, that are processed in machines M1 and M2 (represented by transitions T_1 and T_2), and stored in buffers P_4 and P_6, respectively. Then, they are assembled by M3 (i.e. transition T_3), and processed in M4 (i.e. transition T_4). Finally, M5 (i.e. transition T_5) packages them. During the processing of parts A and B, tool1 (tokens in place P_5) and tool2 (tokens in place P_7) are needed. Also tool3 (tokens in place P_3) have to be used in the three final operations. The machines M1, M2, M4 and M5 are assumed to be reliable and an active redundancy ($n = 3$) is considered for the assembly machine M3 that is assumed to have failure and repair rates $\lambda = 1.5e\text{-}2\ TU^{-1}$ and$\mu = 1e\text{-}1\ TU^{-1}$. The productivity of the workshop is evaluated with the computation of the output flow $X(t, T_5)$ with respect to the number k of pallets and tools : $M_I = (2k\ 2k\ k\ 0\ k\ 0\ k\ 0\ 0\ 3\ 0)^T$. The results obtained with Markov models and SPNs simulation over a time interval of $D = 1000$ TU are summed up in Tables 2 and 3. For $k > 4$, the computational effort becomes heavy because of the large number N of states and the performance evaluation with Markov model analysis is no longer computable.

Table 2: Performance evaluation with Markov models

k	***N***	***X(t, T5)*with Markov model**	**Computational effort (TU)**
1	48	0.29	0.1
2	216	0.61	0.9
3	640	0.93	12
4	1500	1.25	108
5	…	…	…

Simulation with SPNs can be used to overcome the computational limitation with Markov model (Table 3). One can notice that the computational effort remains reasonable even for heavy loaded nets. The simulation error does not exceed 3% for the considered system and is less than 1% for many other cases. The addition of 3 redundant components is enough to reach high availability requirements.

Table 3: Performance evaluation with SPNs

k	**$X(t, T_5)$ with SPN**	**Computational effort (TU)**
1	0.30	0.59
2	0.61	2.0
3	0.94	4.0
4	1.24	6.8
5	1.55	11
10	2.41	30
100	2.63	39

IV. FLUIDIFICATION OF SPNS

A usual limitation encountered with the use of SPNs simulations is the determination of the time horizon *D*. Tide horizons lead to approximation errors and large horizons increase the computation effort. To overcome this difficulty, SPNs can be transformed into timed continuous Petri nets (contPNs) that are compact continuous time models. Such models converge very rapidly to their steady state.

A. *Timed continuous Petri nets*

ContPNs have been developed in order to provide continuous approximations of the discrete behaviors of timed PNs [5,16,3,4]. The marking of each place is a continuous non negative real valued function of time. X_{max} = diag($x_{max\,j}$)$\in(\boldsymbol{R}^+)^{qxq}$ is the diagonal matrix of maximal firing speeds $x_{max\,j}$, j = 1,…q and $X(t, T_j)$ is the firing speed of transition T_j at time t that depends continuously on the marking of T_j input places. The flow through the transition T_j is defined by (12):

$$X(t,T_j)=x_{maxj}.enab(M(t),T_j) \tag{12}$$

with:

$$enab(M(t), T_j) = \min \{M(t, P_k) / w^{PR}_{kj} : P_k \in {}^{\circ}\boldsymbol{T_j}\} \tag{13}$$

where $^{\circ}\boldsymbol{T_j}$ stands for the set of T_j upstream places.

B. *Approximations of SPN with standard Fluidification*

Standard fluidification is the simplest way to transform a stochastic discrete event models into a continuous time one. Standard means that both models have the same structure (i.e. incidence matrices), parameters (i.e. firing rate of the transitions) and initial state (i.e. initial marking). In particular, $x_{maxj} = \mu_j$, $j = 1,...,q$ is used with equations (12) and (13) for standard fluidification[3,4]

An open issue is that standard fluidification of SPNs leads to continuous models so that the steady states of SPNs and contPNs do not coincide in many cases, particularly for non-ordinary PNs or non join-free PNs. As a consequence, availability estimation provided by the steady state of contPNs is different from the one resulting from the analysis of Markov model or from the SPN simulations.

C. *Example*

The example of Figure 9 is considered again and simulated as a contPN. Standard fluidification is used and the results are reported in Table 4.

Table4: Performance evaluation with contPNs

k	$X(t, T_5)$ with contPN	Computational effort (TU)
1	0.33	0.25
2	0.66	0.22
3	1	0.22
4	1.33	0.20
5	1.66	0.22
10	2.61	0.23
100	2.61	0.22

Simulation with contPNs lead to biased results, but the errors do not exceed 8% for $k \geq 10$. One can also notice that the computation effort does not depend on the marking magnitude. Thus, standard fluidificationcan be used to evaluate the performance for heavy loaded net.

V. DISCUSSION AND CONCLUSION

In this paper, a modular modeling with stochastic Petri netsisproposedto represent redundancies in large systems in order to reach high availability requirements. The usual methods (analysis of the corresponding Markov model and SPN simulations) are discussed and fluidification of the discrete event models is presented as an alternative solution that provides good approximations of the mean availability under some specific assumptions (in particular the net is assumed to be heavy loaded).

To conclude, several recent studies have been started to transform SPNs into contPNs that will provide a better approximation of the SPNs behavior in the long run.Markovian and Hybrid Markovian Continuous Petri Nets have been introduced for that purpose [9,10]. These models are continuous time Petri nets including stochastic variables with Poisson pdf. One difficulty is that the resulting models are no longer deterministic. In [11,12], piecewise constant timed continuous PNs have been proposed that are suitable to compute the SPNs steady state in some regions of the marking space. A homothetic approach has been also developed to provide an approximation of the SPNs steady state in the whole marking space [13,14]. Both approaches have been combined with interpolation and classification methods to provide an approximation of the SPN steady state [15].

At this time, the estimation of mean availability for large systems with fluid models remains an open issue that will continue to attract our interest. In particular a supervised combination of SPNs and contPNs in a single hybrid model will be investigated in our future work.

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